NEW TEMPERATURES IN DOMINEERING

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Abstract

Games of No Chance [8] presents a table of known Domineering temperatures, including approximately 40 with denominator less than or equal to 512, and poses the challenge of finding new temperatures. We present 219 Domineering positions with distinct temperatures not shown in *GONC* – including 26 with temperature greater than 3/2 – obtained through exhaustive computational search of several Domineering grid sizes.

1. Background

In the game of *Domineering* [7, 3], players Left and Right alternate placing dominoes on a finite grid of arbitrary size or shape. Left places her dominoes vertically, and Right horizontally. Play continues until a player cannot move; that player is the loser. In spite of its apparent simplicity, many of the properties of combinatorial games can be expressed in Domineering [3].

Analyzing general Domineering positions, *i.e.*, finding their canonical values and temperatures, is difficult. Early work exhaustively analyzed positions of very small size [3]. Since then, inroads have been made for positions with repetitive patterns, such as snakes [13, 9] and $x \times y$ rectangles for particular values of x [1]. There is preliminary work on determining which player can win for rectangular boards of increasingly large dimensions [4, 10], but no comprehensive analysis has been performed on these boards. Because of the complexity of analyzing Domineering positions, much about the values and temperatures of arbitrary Domineering positions remains unknown.

$-1, -\frac{1}{2}, -\frac{1}{4}, -\frac{1}{8}, \ldots$

$0, \frac{1}{8}, \frac{3}{16}, \frac{7}{32}, \ldots$

$\frac{1}{4}, \frac{3}{8}, \frac{7}{16}, \frac{15}{32}, \ldots$

$\frac{1}{2}, \frac{3}{4}, \frac{7}{8}, \frac{15}{16}, \ldots$

$1, 1\frac{1}{8}, 1\frac{3}{16}, 1\frac{1}{4}, 1\frac{5}{16}, 1\frac{11}{32}$

Table 1: Previously known temperatures in Domineering as presented in [8].

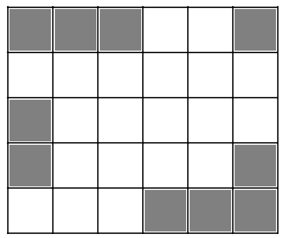

Figure 1: A Domineering position with several left and right options.

Games of No Chance [8] presents a table of known Domineering temperatures, seen in Table 1, including approximately 40 with denominator less than or equal to 512. *GONC* then poses the challenge of finding new temperatures. Since the publication of *GONC*, Gabriel Drummond-Cole discovered a position of temperature 2 and several others with temperatures between 3/2 and 2 [5]; however, we know of no other published results presenting new Domineering temperatures.

A computational search technique was successfully applied to other games such as Fox and Geese [12]; we are not aware of previous work that applies exhaustive search to Domineering.

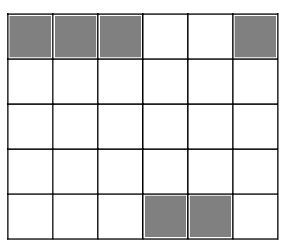

Figure 2: A Domineering position whose canonical form has more than 20,000 characters.

2. New Temperatures

We modified version 0.3 of Aaron Siegel's CGSuite tool [11] to enumerate all Domineering positions that can be fit into boards of dimensions 5x6, 4x8, 3x10, and 2x16, recording positions with unique temperatures[1]. The computation resulted in the discovery of 219 new temperatures, including 26 with temperature greater than 3/2, all with a denominator of 512 or less. These new temperatures are particularly exciting since they occur on positions small enough to arise in tournament play; some more esoteric temperatures found in previous work occurred on less common positions of larger dimensions. We also found 40 additional temperatures in *generalized Domineering*, in which the position is not necessarily reachable through gameplay from an empty rectangular grid.

Our search of over 10 billion positions found no position with temperature greater than 2. Elywn Berlekamp has long sought a proof of a maximum temperature in Domineering [2], and while our technique cannot provide such a proof, it lends credence to the current belief that 2 is the largest possible temperature in Domineering.

Among the grids that we searched, we have found all temperatures between 0 and 2 with denominator 16, all but two with denominator 32, all but 15 with denominator 64, and so on[2]. An open question is whether every dyadic rational temperature between 0 and 2 is achievable given a sufficiently large Domineering position.

3. Additional Analysis

Our exhaustive computational search complements previous approaches to analyzing Domineering positions in its ability to handle arbitrary positions, rather than just those that fit some pattern. Previous work exploited patterns in empty rectangular boards [1, 4, 10] or repetitive non-rectangular patterns [13, 9] to generalize and ease the analysis process. Our technique finds and evaluates irregular Domineering positions that do not easily fit into such patterns and seem quite difficult to analyze by hand – but that still have interesting values and temperatures.

For example, we encountered millions of distinct Domineering *values* in our search for new temperatures, 80% of which had multiple options. Figure 1 presents an example of one such interesting Domineering position[3], with eight canonical left options and seven canonical right options. Note that left only has 13 choices for a move, and right only 15; over half are canonical choices for play. It is rare to find so many undominated options in such a small

[1]The modifications are available as a CGSuite plugin; email the authors if interested.

[2]Gabriel Drummond-Cole has since found positions with the two temperatures with denominator 32, 61/32 and 63/32. Our search did not find these positions because they do not fit into the board sizes we searched[6].

[3]The canonical form of this position was too large to print; it can be found by entering the position into CGSuite [11].

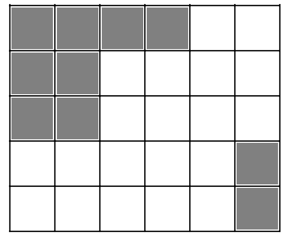

Figure 3: A Domineering position whose value is $+_{5/2}$.

position. We also found many positions with large and complex canonical forms, the largest of which (seen in Figure 2) had more than 20,000 characters.

We have also discovered a variety of positions with tiny and miny values; few positions with such values are known in Domineering [2]. The values our search encountered include $+_{1/4}, +_{1/2}, +_{3/4}, +_{1}, +_{3/2}, +_{2}, +_{5/2}$ and their starred equivalents (e.g. $+_{1/2*}$), as well as other more complex tiny values. Figure 3 shows a position with value $+_{5/2}$.

Our exhaustive search can easily be modified to output any information about the discovered positions, such as thermographs, chilled values of positions, etc.

Section 4 presents the standard Domineering positions with new temperatures that we found, and Section 5 presents the generalized positions.

We would like to thank Prof. Elwyn Berlekamp for his support of this project, as well as the anonymous referee for helpful suggestions.

4. Standard Positions

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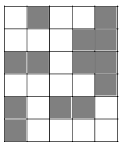
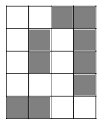
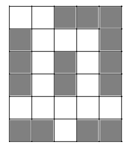
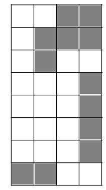
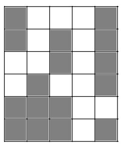
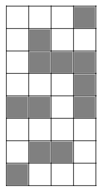
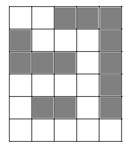
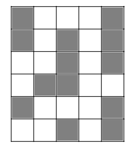
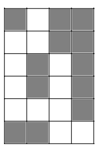
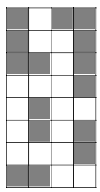
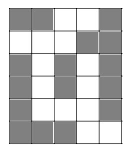
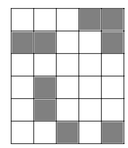
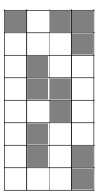
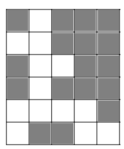
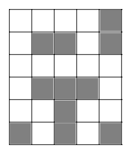
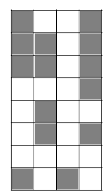
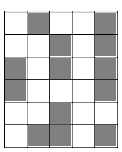
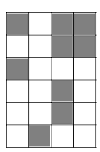
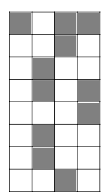
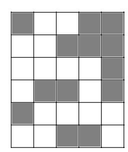
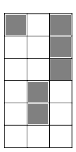
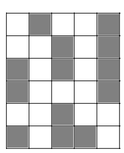
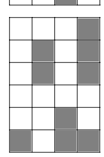
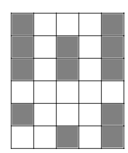
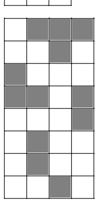
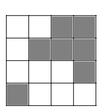
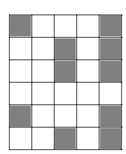
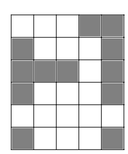
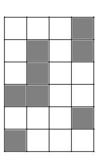
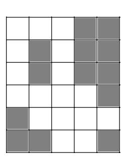
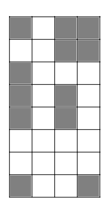
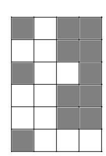
	1/32		1/16		3/32		7/64
	5/32		13/64		9/32		19/64
	5/16		21/64		11/32		23/64
	25/64		13/32		27/64		29/64
	33/64		17/32		69/128		35/64
	9/16		37/64		19/32		39/64
	79/128		5/8		81/128		41/64
	21/32		43/64		87/128		11/16

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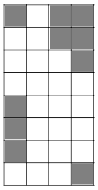
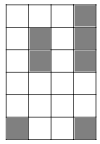
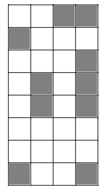
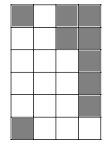
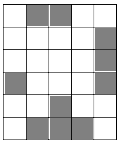
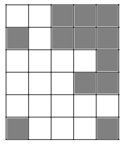
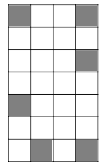
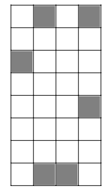
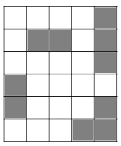
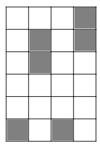
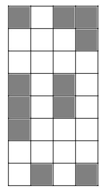
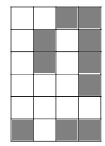
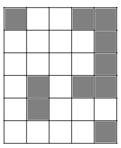
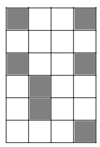
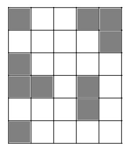
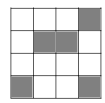
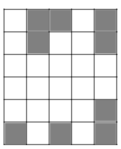
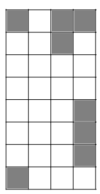
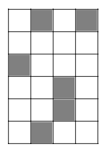
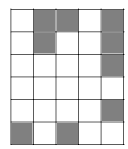
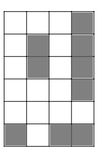
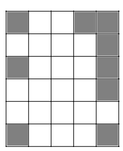
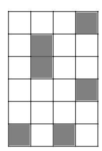
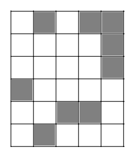
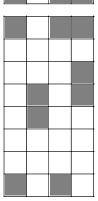
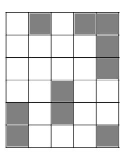
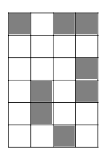
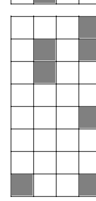
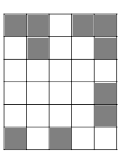
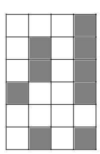
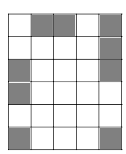
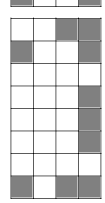
	89/128		45/64		91/128		23/32
	93/128		47/64		95/128		193/256
	97/128		49/64		99/128		25/32
	101/128		51/64		103/128		13/16
	105/128		211/256		53/64		107/128
	27/32		109/128		55/64		111/128
	225/256		113/128		57/64		229/256
	115/128		29/32		117/128		235/256

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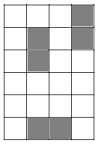
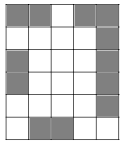
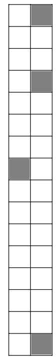
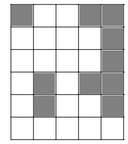
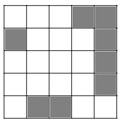
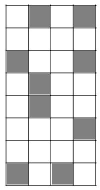
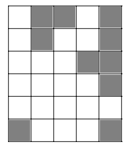
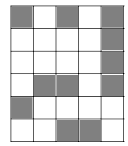
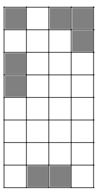
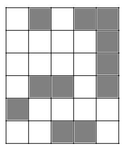
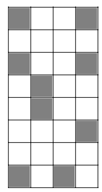
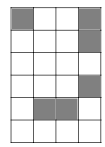
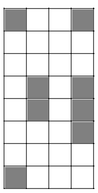
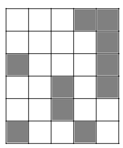
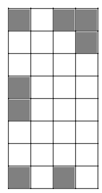
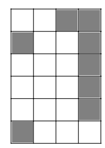
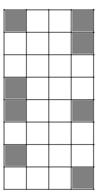
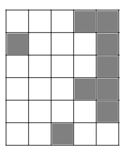
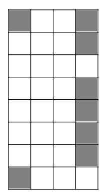
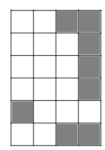
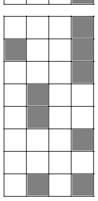
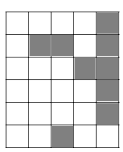
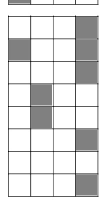
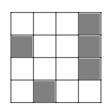
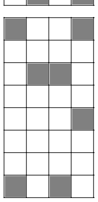
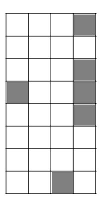
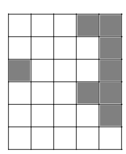
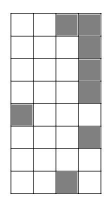
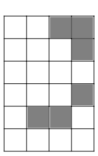
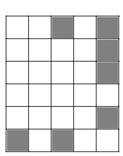
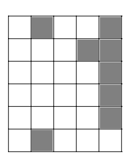
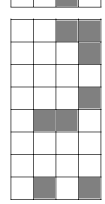
	59/64		119/128		239/256		121/128
	61/64		245/256		123/128		125/128
	257/256		129/128		259/256		65/64
	261/256		131/128		263/256		33/32
	265/256		133/128		267/256		67/64
	269/256		135/128		271/256		17/16
	273/256		547/512		137/128		275/256
	69/64		277/256		139/128		279/256

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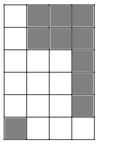
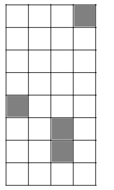
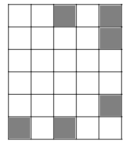
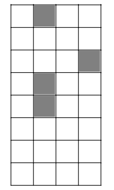
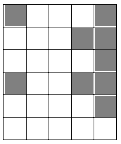
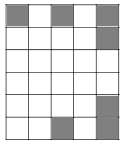
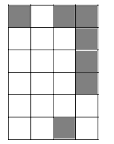
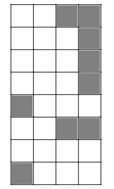
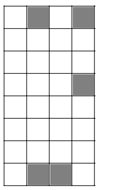
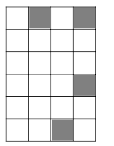
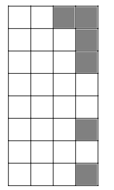
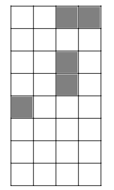
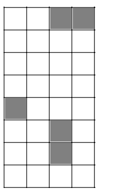
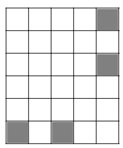
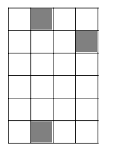
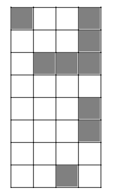
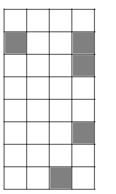
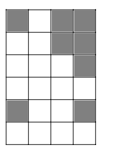
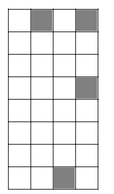
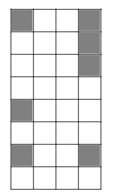
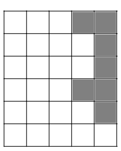
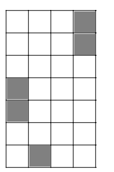
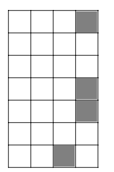
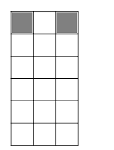
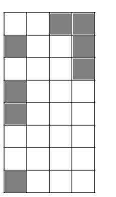
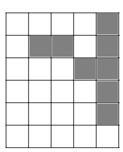
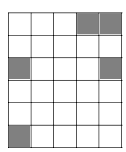
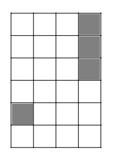
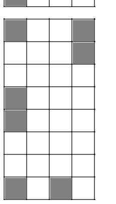
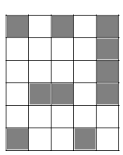
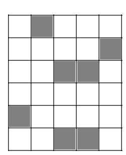
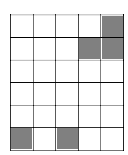
	35/32		561/512		281/256		563/512
	141/128		283/256		71/64		285/256
	571/512		143/128		287/256		575/512
	577/512		289/256		145/128		291/256
	583/512		73/64		585/512		293/256
	147/128		295/256		591/512		37/32
	297/256		149/128		299/256		75/64
	301/256		151/128		303/256		305/256

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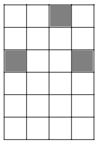
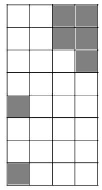
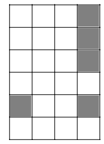
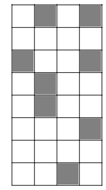
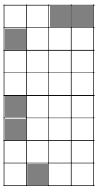
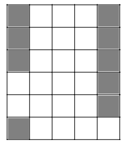
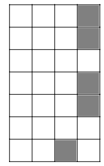
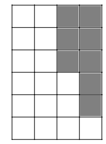
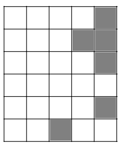
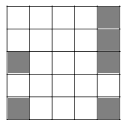
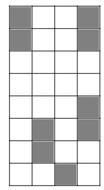
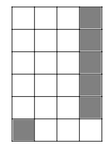
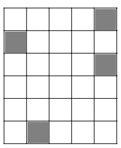
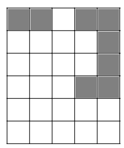
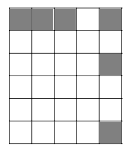
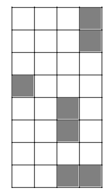
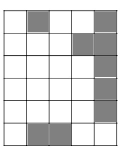
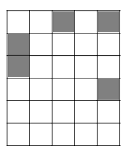
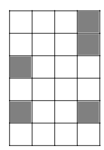
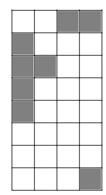
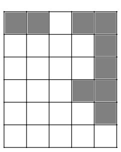
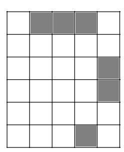
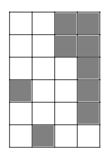
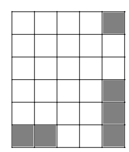
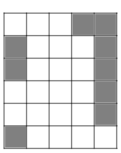
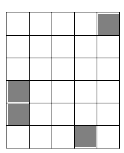
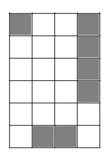
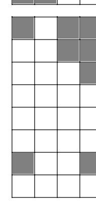
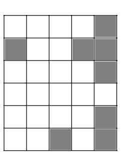
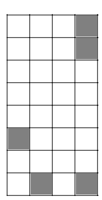
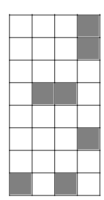
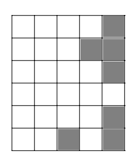
	153/128		307/256		77/64		309/256
	619/512		155/128		311/256		39/32
	313/256		157/128		315/256		79/64
	317/256		159/128		319/256		321/256
	161/128		323/256		81/64		325/256
	163/128		327/256		41/32		329/256
	165/128		331/256		83/64		333/256
	167/128		335/256		337/256		169/128

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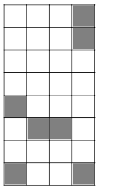
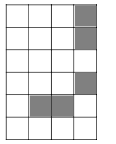
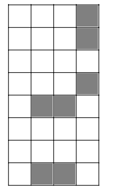
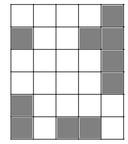
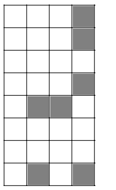
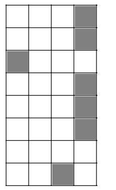
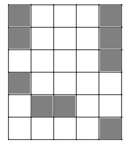
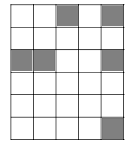
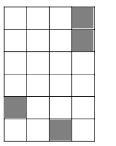
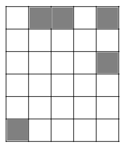
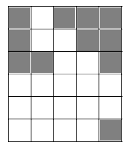
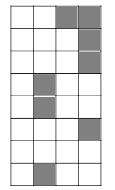
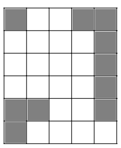
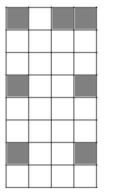
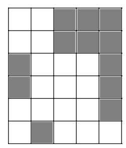
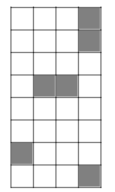
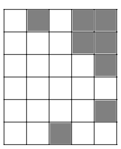
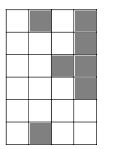
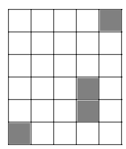
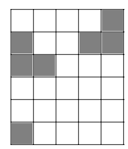
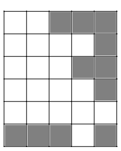
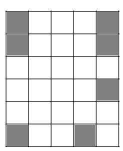
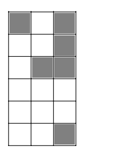
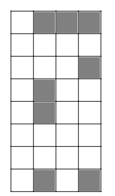
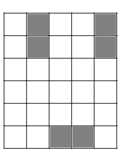
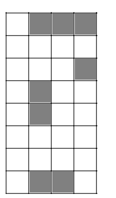
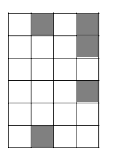
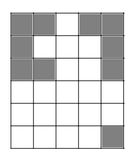
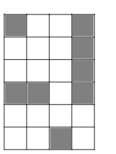
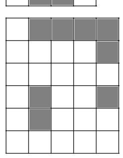
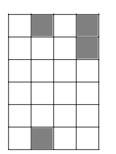
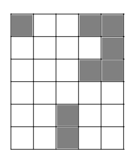
	339/256		85/64		341/256		171/128
	343/256		345/256		173/128		347/256
	87/64		349/256		175/128		351/256
	177/128		355/256		89/64		357/256
	179/128		45/32		361/256		181/128
	91/64		183/128		23/16		369/256
	185/128		371/256		93/64		187/128
	47/32		189/128		95/64		191/128

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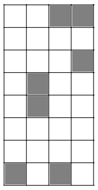
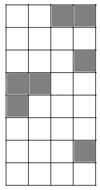
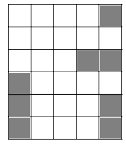
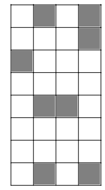
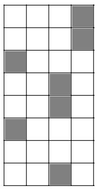
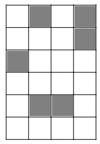
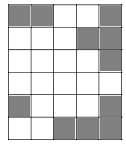
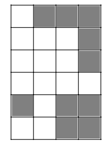
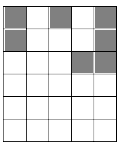
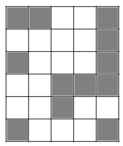
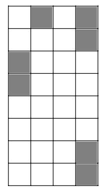
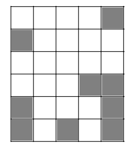
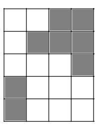
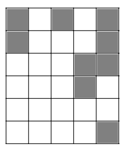
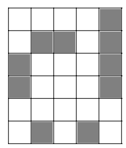
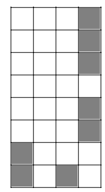
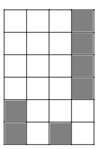
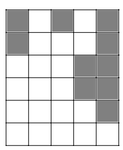
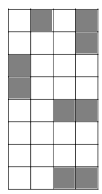
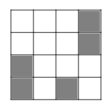
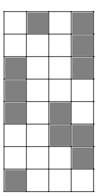
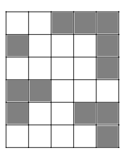
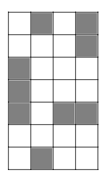
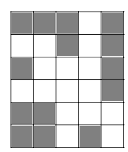
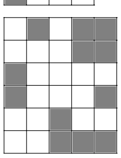
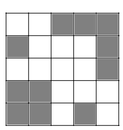
	385/256		193/128		97/64		195/128
	391/256		49/32		99/64		25/16
	101/64		51/32		205/128		103/64
	13/8		105/64		53/32		107/64
	27/16		55/32		111/64		7/4
	57/32		29/16		59/32		15/8
	31/16		2				

5. Generalized Positions

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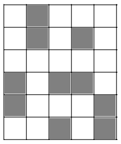
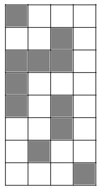
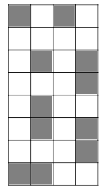
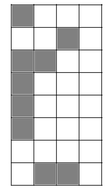
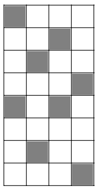
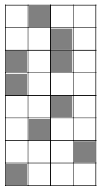
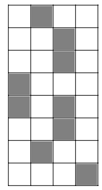
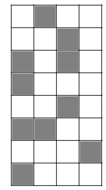
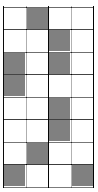
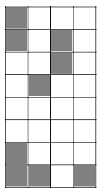
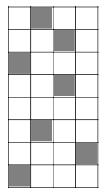
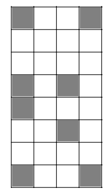
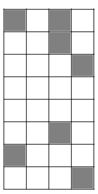
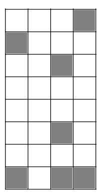
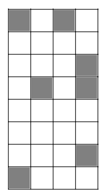
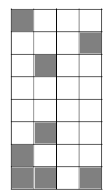
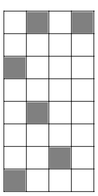
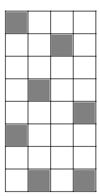
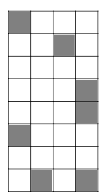
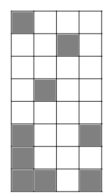
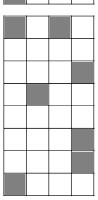
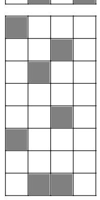
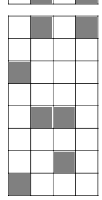
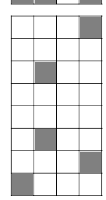
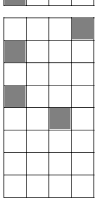
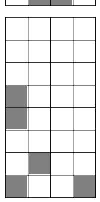
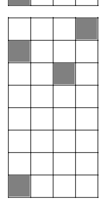
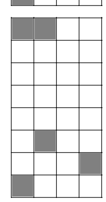
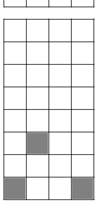
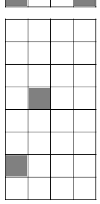
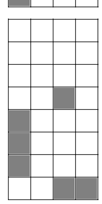
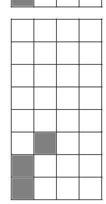
	1/64		17/64		57/128		65/128
	71/128		73/128		75/128		77/128
	83/128		85/128		201/256		215/256
	219/256		221/256		223/256		227/256
	237/256		241/256		243/256		247/256
	251/256		253/256		255/256		541/512
	565/512		573/512		581/512		601/512
	603/512		617/512		353/256		359/256

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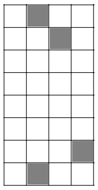
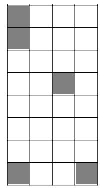
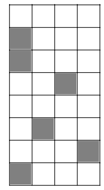
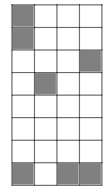
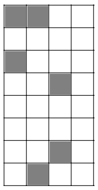
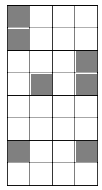
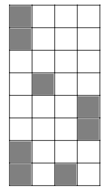
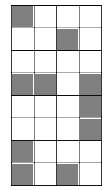
	363/256		365/256		383/256		197/128
	199/128		211/128		109/64		113/64

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[1] Elwyn Berlekamp, *Blockbusting and Domineering*, JCTA **49** (1988), 67-116,

[2] Elwyn Berlekamp, personal communication, 2004.

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