The Complexity of Andersen's Analysis in Practice

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SAS 2009

Andersen's Analysis

- **Definition (almost)**
  precise flow- and context-insensitive points-to analysis
- Presented by Andersen in 1994
  – Similar predecessors, e.g., 0-CFA [Shivers88]
- Worst-case complexity ***nearly cubic*** ($O(N^3 / \log N)$)
- Early implementations didn't scale
  – Approximations developed [Steensgaard96,Das00]

Scaling Andersen's Analysis

Online Cycle Elimination
[FFSA98,HT01,HL07]

Type Filters
[LH03]

Preprocessing
[RC00,HL07]

Shared Bitsets / BDDs
[HT01,BLQHU03,ZC04,WL04]

Projection Merging
[SFA00]

And More!
[next talk]

Impressive Scalability: 1M C LOC [HL07] or 500K Java bytecodes [WL04] in under 10 minutes

Why Does Andersen's Scale?

Our work, for Java

Possibility 1: Reduced Constant Factors

Nearly cubic behavior remains in practice

Possibility 2: Real Programs Easier

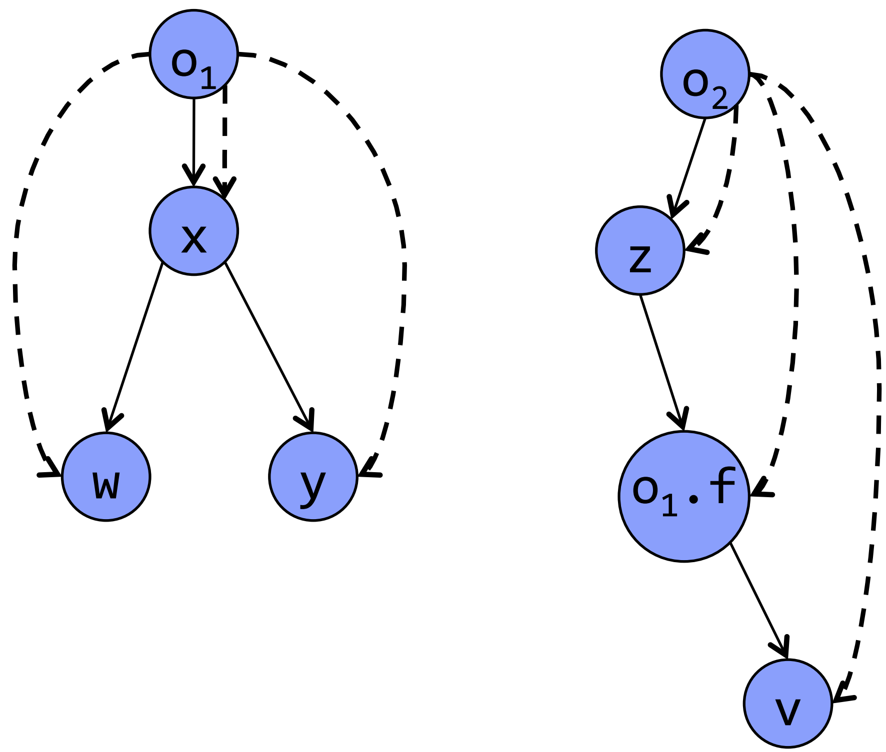
Program structure enables *subcubic scaling* in practice

Key Results

- Andersen's analysis is **$O(N^2)$ for *k*-sparse programs**
- For Java, *k*-sparsity through **types** + **encapsulation**
  - Structure makes analysis easier than for C
- **Empirical validation**
  - Benchmarks from 176-2225K bytecodes
  - Showed *k*-sparsity and quadratic scaling

Background: Andersen's as Dynamic Transitive Closure

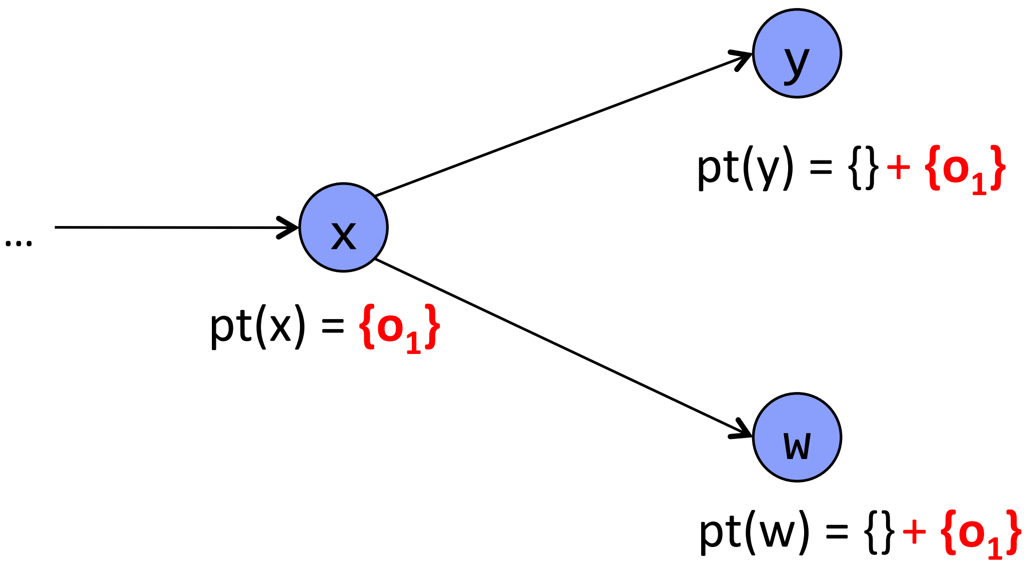
```
1: x = new Obj();
2: z = new Obj();
3: w = x;
4: y = x;
5: y.f = z;
6: v = w.f;
```



Complexity of chaotic worklist algorithm: **$O(N^4)$**

Background: Difference Propagation

“Standard” Propagation



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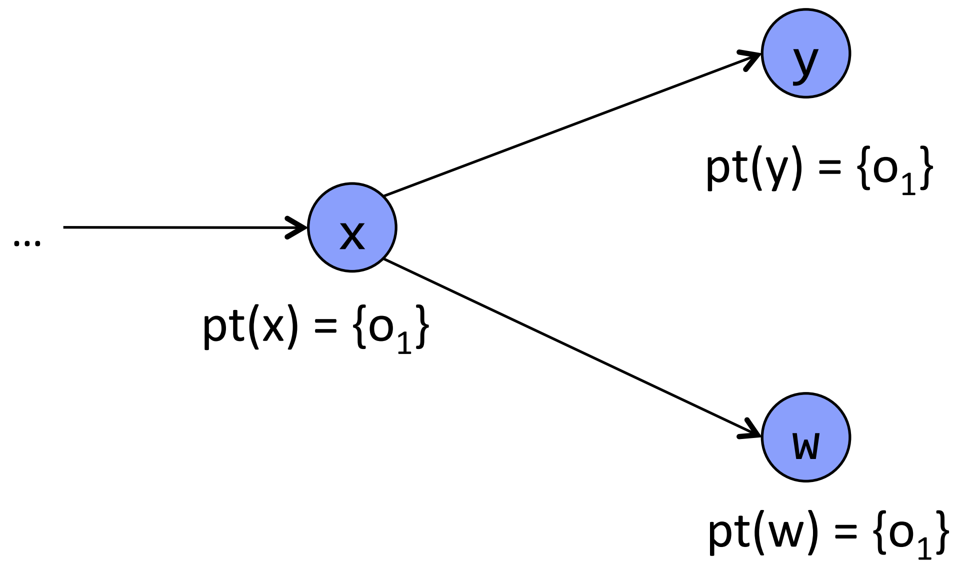

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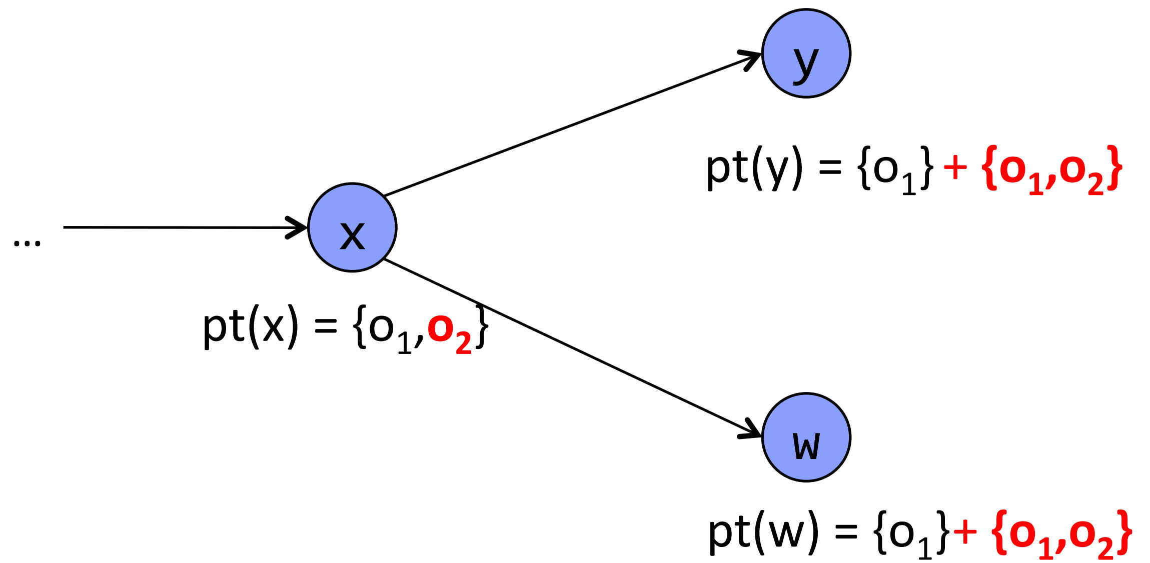

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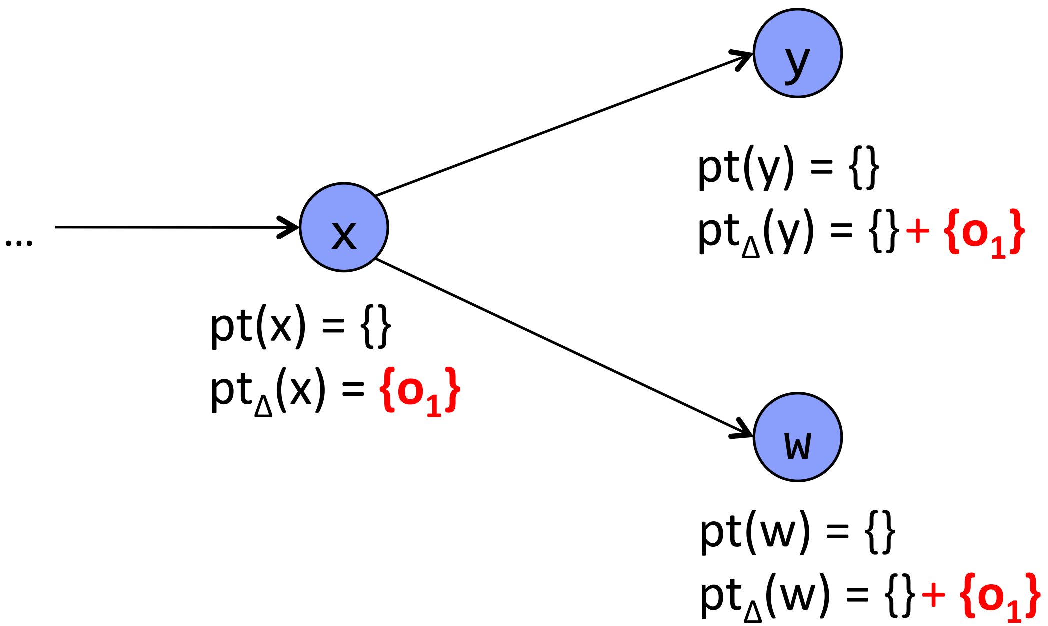

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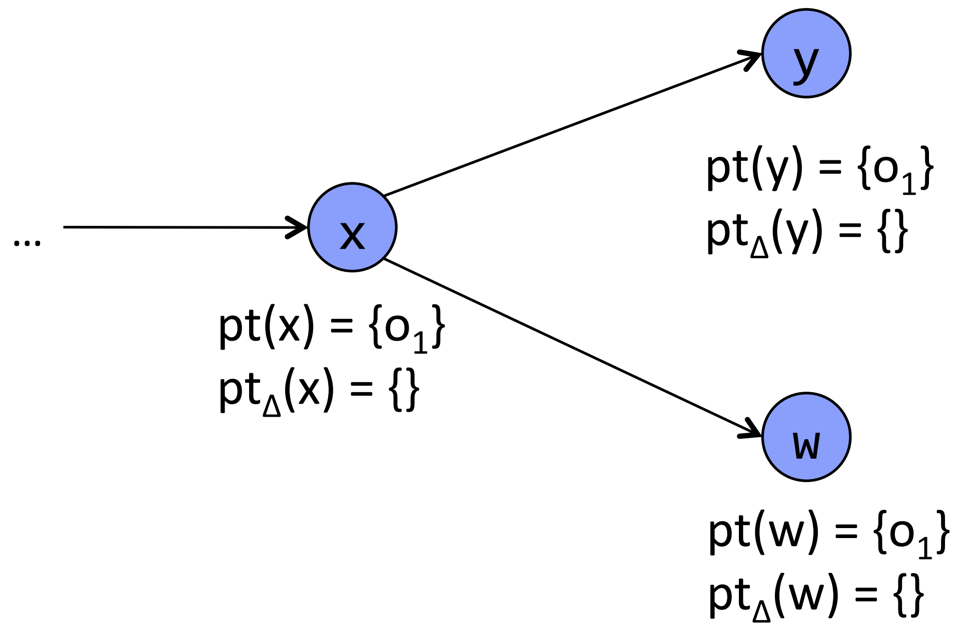

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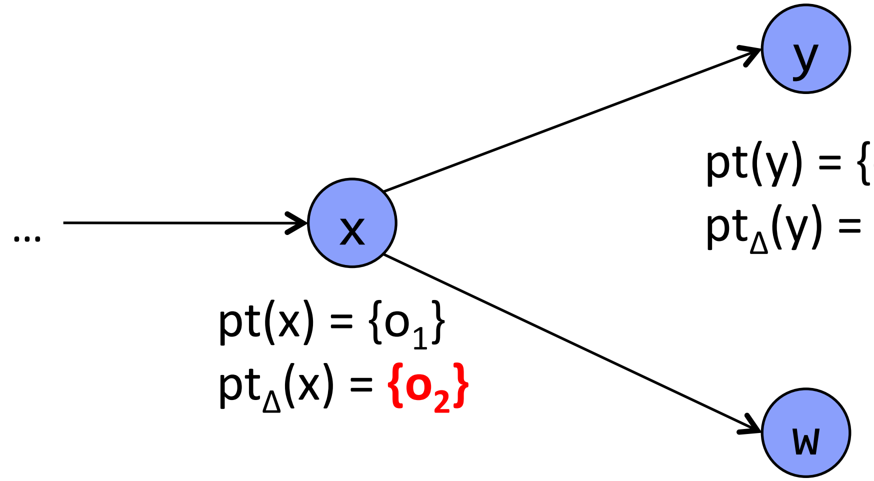


$pt(x) = \{o_1\}$

$pt_{\Delta}(x) = \{o_2\}$

$pt(y) = \{o_1\}$

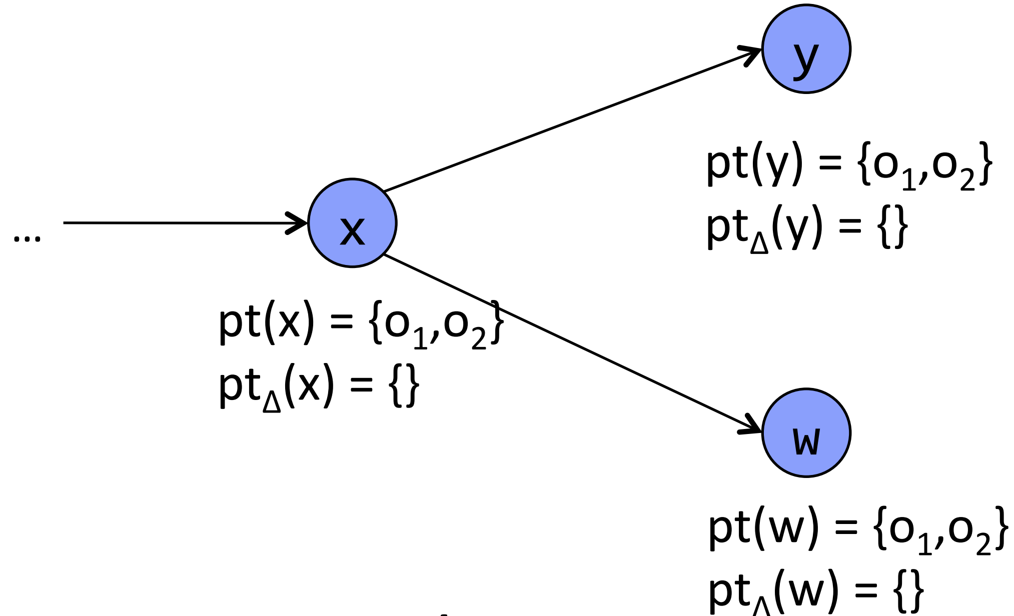
$pt_{\Delta}(y) = \{\} + \{o_2\}$

$pt(w) = \{o_1\}$

$pt_{\Delta}(w) = \{\} + \{o_2\}$

Background: Difference Propagation

Difference Propagation

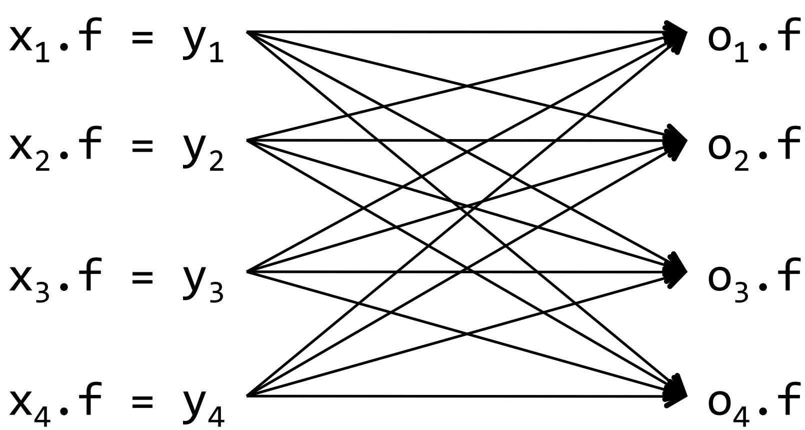


- Guarantee: loc propagated ***at most once per edge***
- DTC + difference propagation complexity: $\mathbf{O(N^3)}$ [Pearce05]

k-sparse programs

- Def: num. of graph edges ≤ *k* * N *at termination*, *k* constant
- Complexity for *k*-sparse programs: **$O(N^2)$**
  - Linear number of edges, linear work per edge (via diff. prop.)
  - Must also count edge adding work; see paper for details

Non-*k*-sparse graph



Java and *k*-sparsity: *strong types*

```
class A { int f; }
class B { int g; }

A a = new A();
a.g = 5; // compile error
```

Key benefits: **few fields per object, no aliased fields**

- Limits number of object field nodes created
- Exploited in previous work [SGSB05,SB06]

Unlike C: no structure casts

Java and *k*-sparsity: *encapsulation*

```
class C {
  // encapsulated
  private int state;
  int getState() { return this.state; }
  void setState(int i) { this.state = i; }
}
```

Benefit: **few accesses per field**

– Limits number of closure edges

– Tradeoff: possibly worse precision (context insensitivity)

Unlike C: no * operator

Threats to *k*-sparsity

- Dynamic dispatch
  - # of targets at call sites may increase with program size
  - Haven't observed in practice; on-the-fly call graph helps
- Arrays
  - `y = x[0];` ➔ `y = x.arr;`
  - Same `arr` field for all array types (due to subtyping)
  - # of accesses of `arr` increases with program size (like * in C)
  - Observed some blowup in one benchmark

Experiments

Implementation

T. J. Watson Libraries for Analysis (WALA)

http://wala.sf.net

Benchmarks

Dacapo 2006-10-MR2 + Apache Ant

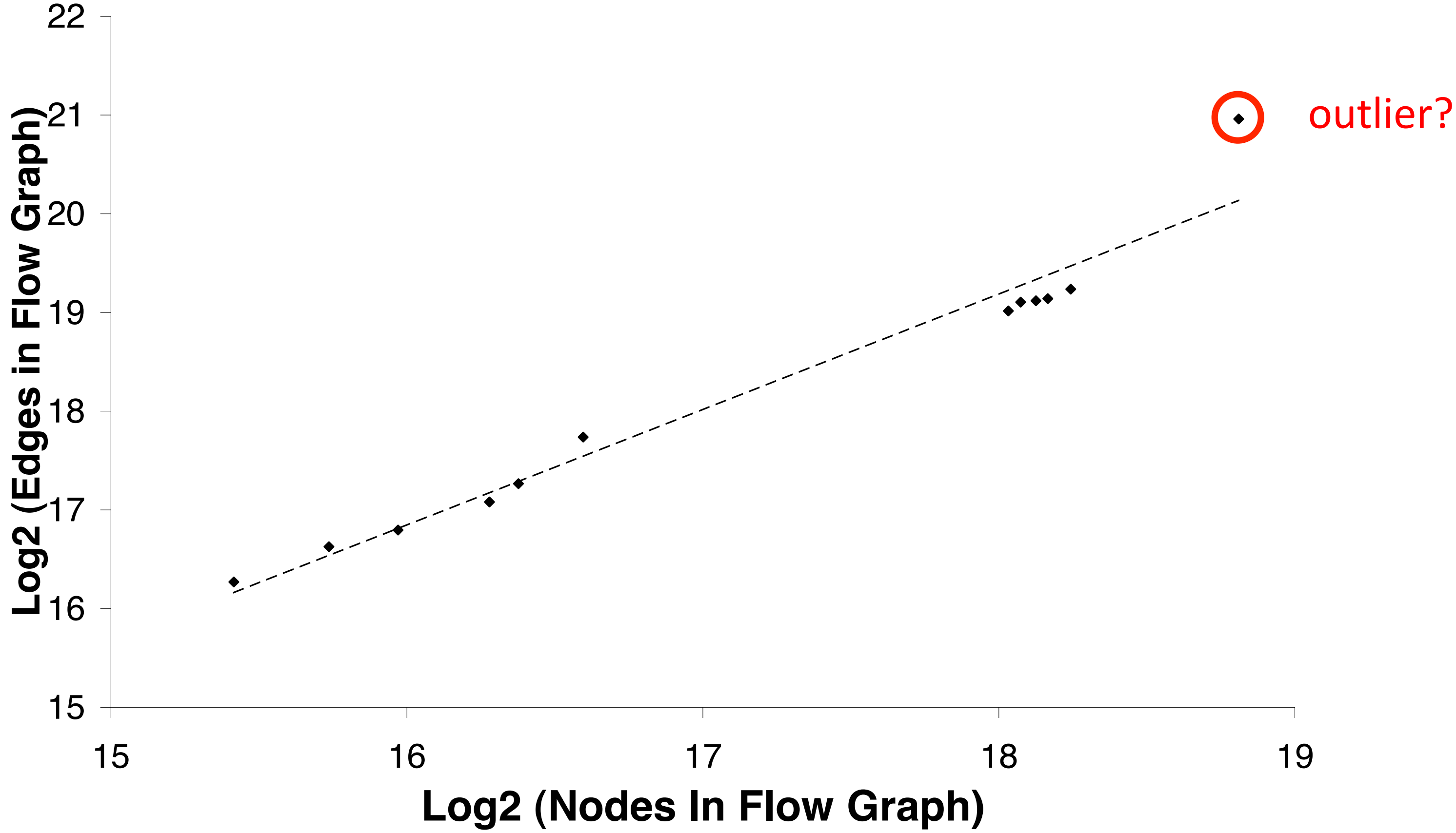
IBM Java 1.6.0 libraries

176-2225K bytecodes (largest published)

Questions

1. Are programs *k*-sparse?
2. Is quadratic scaling observed?
3. How tight is quadratic bound?

Are programs *k*-sparse?



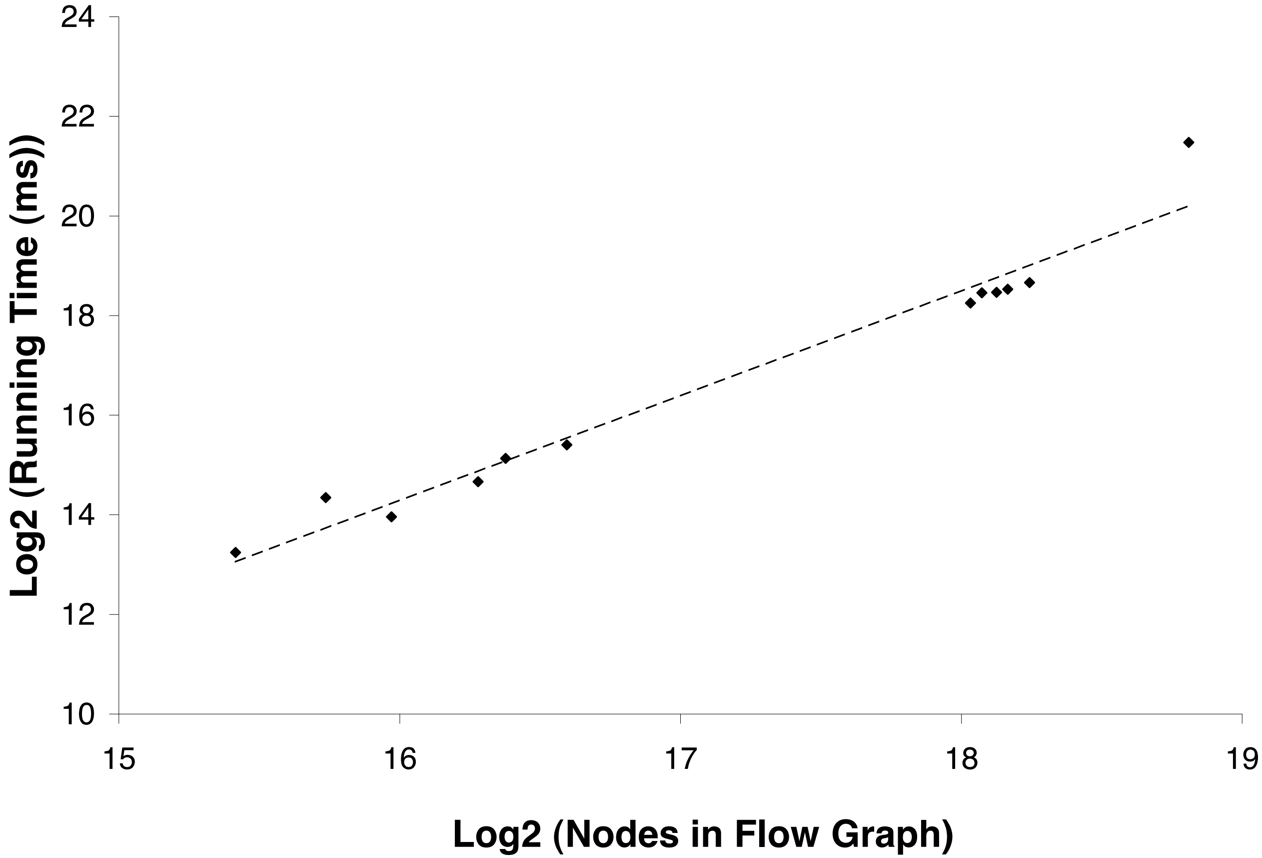
$E = N^{1.17}$ ($N^{1.05}$ excluding "outlier");

$k \leq 4.44$

fop “outlier” benchmark

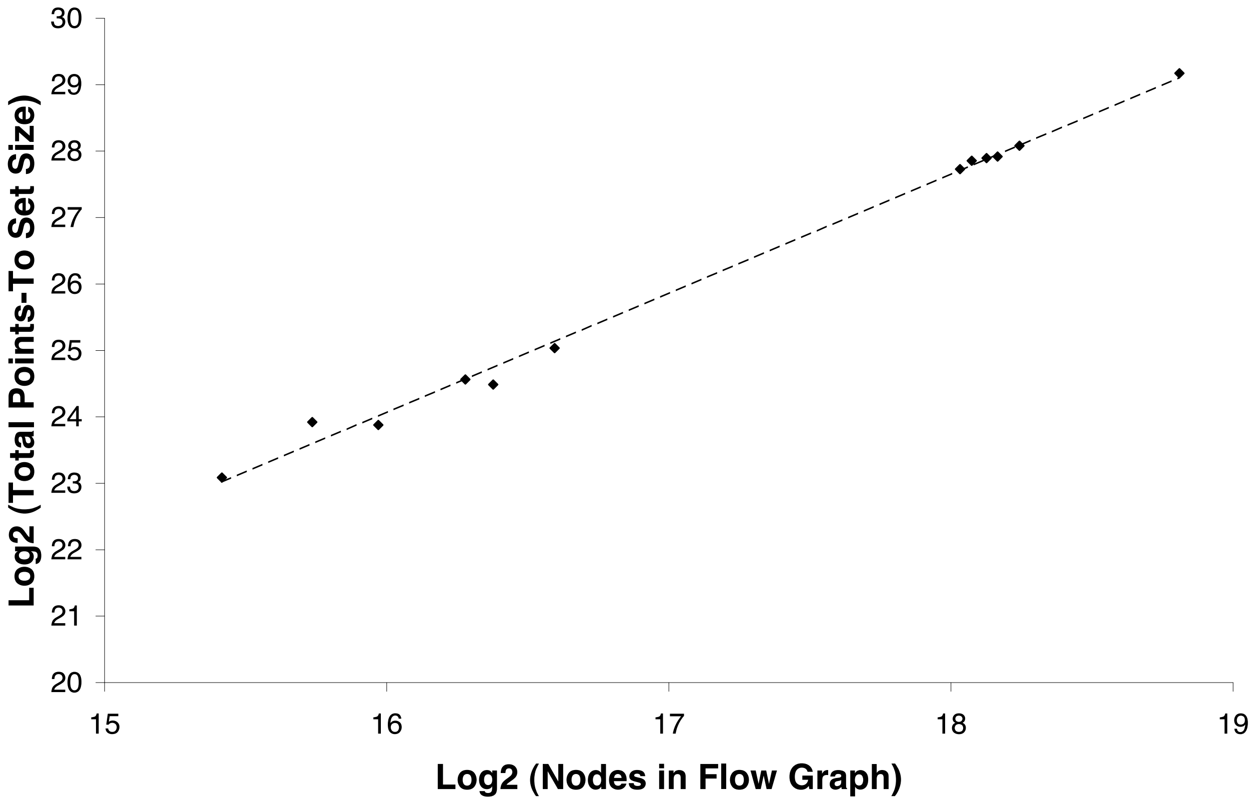
- Extensive use of library array manipulation routines
  - E.g., from `java.util.Arrays`
- Arrays + context-insensitive handling of routines pollutes results
- **Lesson**: targeted context sensitivity could improve both precision and performance
  - Especially for array-handling routines

Is quadratic scaling observed?



Time = $N^{2.10}$ ($N^{1.92}$ excluding “outlier”)

How tight is quadratic bound?



Total points-to size = $N^{1.79}$

Other Factors

- Standard techniques can provide constant-factor speedups
  - Bit vector parallelism, type filters, on-the-fly call graph, preprocessing, cycle elimination
- Space considerations very important in practice
  - May not want exhaustive use of delta sets
  - BDDs / shared bit sets reduce space but complicate running time analysis
- Detailed discussion in paper

Open questions

- What about other languages?
  - Some evidence that C programs are *not* *k*-sparse [PKH03]; may explain greater importance of cycle elimination
  - Result translates to 0-CFA; are functional programs *k*-sparse?
- Time complexity for BDDs / shared bit sets?
- Is tighter bound possible?
  - Demand-driven analysis may have less required output
- Does *k*-sparsity help other analyses?

Conclusions

- Andersen’s is quadratic for *k*-sparse inputs
- Realistic Java programs are *k*-sparse
  - **Strong typing**
  - **Encapsulation**
- Explains (partially) the scalability of Andersen’s for Java in practice; no cubic bottleneck!

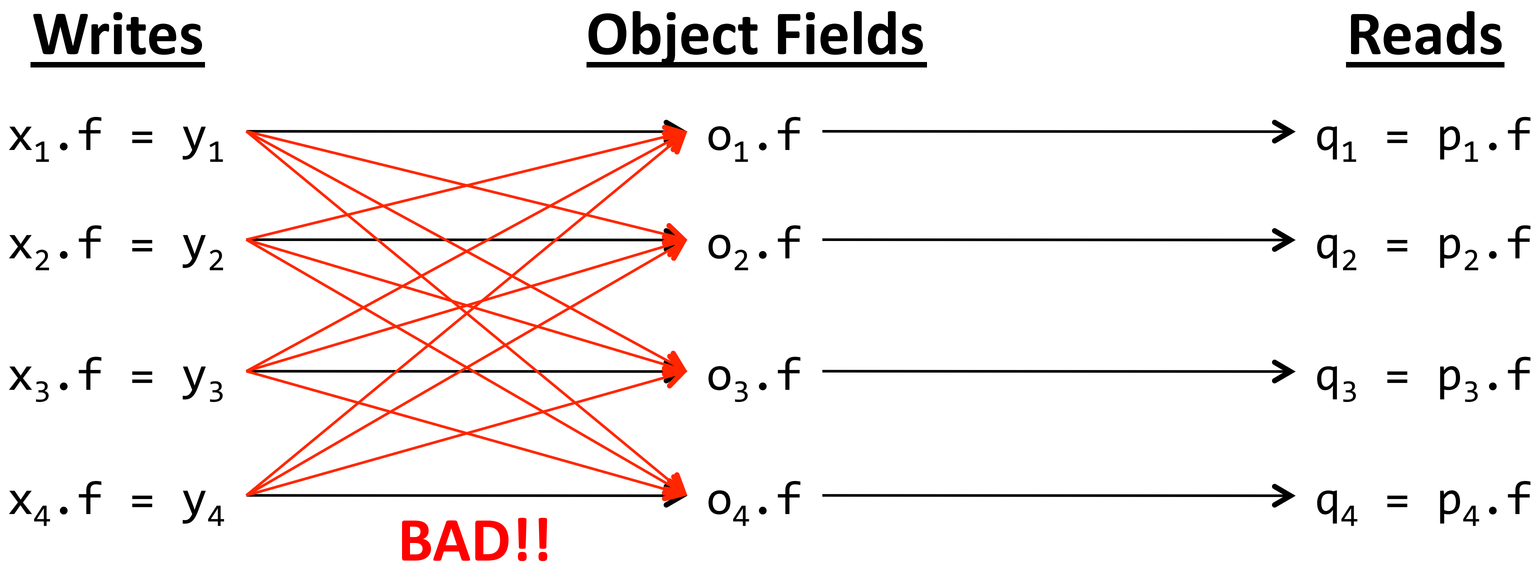
Thanks!

Formulation of Andersen's Analysis

Statement	Constraint	
`i: x = new T()`	$o_i \in pt(x)$	[New]
`x = y`	$pt(y) \subseteq pt(x)$	[Assign]
`x = y.f`	$\dfrac{o_i \in pt(y)}{pt(o_i.f) \subseteq pt(x)}$	[Load]
`x.f = y`	$\dfrac{o_i \in pt(x)}{pt(y) \subseteq pt(o_i.f)}$	[Store]

k-sparse programs

Final number of edges <= *k* * N



Complexity for *k*-sparse programs: **$O(N^2)$**

- Linear work per edge (diff. prop.), linear number of edges
- Also edge adding work; see paper for details